THE BEAUTY AND SPIRITUALITY OF MATHEMATICS: A REVIEW ESSAY

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Introduction

«*The intention of mathematics teaching is to promote the learning of mathematics*» is a statement that will be challenged by few. This book gives portraits of students and focuses on consciousness and inspiration, values and experiences in mathematics – it tries to uncover why some students find inspiration in the subject of mathematics.

The Danish philosopher Sören Kierkegaard has once said that «*geniuses are like thunderstorms – they go up against the wind, they terrify people, and they cleanse the air*»[1]. The German author and artist Johann Wolfgang von Goethe (1749–1832) has supposedly said that *everything has been thought of before; the task is to think of it again in ways that are appropriate to one's current circumstances.* Mathematics is an enormous adventure of ideas, and the history of mathematics is reflecting some of the noblest ideas and thoughts of countless generations (Struik 1987). These ideas emerged when the time was right, and both the ideas and the context in which they emerged are useful knowledge in addition to mathematical content knowledge.

A mathematician like G. H. Hardy (1877–1947) is inevitable in a book about aesthetics in mathematics. His famous little pearl of a book, *A Mathematician's*

[1] Genier er som tordenvær – de går mot vinden, de forfærder menneskene og de renser luften

Apology (Hardy 1940), is a constant source for good quotations for any lecturer, and there he claims that very few has the ability to become good mathematicians. Hardy also writes that

> *«The mathematician's patterns, like the painter 's or the poet's, must be* beautiful*; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.» (Hardy 1940: 85)*

Reuben Hersh (1999: 29) mention similar ideas when he writes that «*elegance is more common in mathematical theories than in philosophical ones*». Beauty and elegance is of course to be desired in philosophy, but it is not a prerequisite. Mathematics is precise, while philosophy cannot be. There are proofs in mathematics that is beautiful just because of their elegance – some classic examples are Euclid's proof that the number of primes is infinite, and Pythagora's proof that $\sqrt{2}$ is irrational.

Isaac Newton gave credit to the the accumulated work of his predecessors for his foresight in his development of calculus – «*if I have seen further it is by standing on the shoulders of giants*». The mathematician Norbert Wiener is one of the many scientists who has broken the bonds of traditions, and has created new fields to explore. Wiener is honoured as the one who introduced cybernetics, and of him it is also said that «*he just saw further than the rest of us*». The traditional description of mathematics has been as the science of number and shape. Arithmetic, measurement, algebra and geometry has traditionally been the stuff of school mathematics, but there are deeper ideas that helps the growth and development of mathematics. One may think of specific mathematical structures like numbers, algorithms, ratios, shapes or functions, and in a similar fashions may one think of mathematical attributes, actions, abstractions, attitudes, behaviours and dichotomies like discrete vs. continuous, finite vs. infinite etc. (Steen 1990). This shows the complexity of mathematics and illustrates the fact that new mathematical ideas may be developed from various stands. School mathematics picks traditionally few of these stands, but not since the days of Newton and Leibnitz has mathematics changed so much as in recent years. So we need to give the students and pupils the opportunity to *stand on the shoulders of those giants who saw further than the rest of us.*

This book is trying to discover *why* some becomes inspired by mathematics, and what the *spiritual inspirations* are. Klaus Witz' ideas are to be found in the commentaries to the interviews. The chapters 2 through 7 contains the research, and chapter 8 starts with the rhetorical question «*What emerges from these four portraits?*» . The research then comes together in chapter 9 where we find the philosophy of education.

The structure of the book

The bottom line in the interviews is «*why* are these students interested in mathematics, and *what* drives them». The motivating power while reading the book is what we can learn from the book ad the interviews, and what values Klaus Witz is communicating. I will here try to pass on some personal remarks and experiences to the various chapters.

Forewords

The first philosophically angled foreword is written professor Ubiratan D'Ambrosio from São Paulo, and he starts by posing the question «*What is mathematics, really?*». He also refers to the wizdom of *Upinashads*, meaning the inner or mystic teaching, where groups of pupils sit near the teacher to learn from him the secret doctrine. Mathematics is a common knowledge, and in a socio-cultural perspective one may ask *how is mathematics internalized*? The driveforces are inner voice, inner revelation and inner peace.

The second foreword is written by Kenneth J. Travers, professor of mathematics education, and is more didactically angeled. He write that the Klaus Witz has found i necessaryto develop a new methodology, namely «*the participant as ally – essentialist portraiture*».

And finally, the last foreword, written by professor Jerry Uhl, a teacher in mathematics. As such he is asking a lot of interesting questions, and he is, not surprisingly, referring to chapter 8 as the most important in the whole book. This book may throw some light over the question why such a small persentage of students chooses to study mathematics , even if they spend hours every week reading it. This book contains case studies of individual students – what Klaus Witz calles *portraits*.

Introduction

The introduction has the subtitle «*The Mystery of Mathematics as Seen in the Experience of Undergraduates*». Klaus Witz describes here his own background – after a PhD in mathematics he worked in a mathematics department in a university where he became convinced that many students where inspired my by the subject. Mathematics has since the Antiquity been wrapped in mystery – what is mathematics really, what is mathematical entities, and how does mathematics relate to the universe and to the reality? Professor Witz continues that an appropriate way to investigate this is to talk to the students. They started to see something in mathemstics that they had never seen before.

The introduction goes through each of the chapters of the book, and especially chapter 8, in a very thorough way. In chapter 8 the question «*what greater coherence emerges from these studies*» is posed, and chapter 9 tries to put the enlightenment from chapter 8 in a greater perspective.

Chapter 1 – an understanding of Klaus Witz' ideas

Why do students focus on particular subjects, why do they decide to pursue that subject? It has always been a mystery *what* mathematics really is, and what is it about this subject that motivates and gratifies some. *Bildung* has always been a motivating power when it comes to education in German countries, and that is about forming the individual. Witz gives in this connection thorough descriptions of the terms *Bildungsinhalt* and *Bildungsgehalt*.

Take for instance the law for free fall

$$x = -\frac{1}{2}\frac{g}{m}t^2 + d$$

When a student understands this law, he or she will be stricken by the simplicity and beauty at a deeper level – that is *Bildungsgehalt*. The only reasons to pursue the subject of mathematics is *beauty* and *truth*. Mathematics has at all times been attached to *beauty* and *truth*! The famous hungarian mathematician Paul Erdös (1913–1996) is quotet to have said that «*man can learn everything about physics and biology, but definitively not about mathematics, because mathematics is itself infinite*» (Witz 2007: 21).

A spiritual element in mathematics may be the reason that many finds gratification in the subject – something that one may find in mathematics, and not in any other subject. This is a naturalistic qualitative understanding of humans versus a psychometric understanding. Witz asserts that there is no research in mathematical knowledge as a natural language. Does some become good mathematicians because the surroundings encourage them?

The first american to receive a doctorate at Göttingen, Edward Everett (1794–1865), wrote truthfully about the ideas of mathamatics that «*in the pure mathematics we contemplate absolute truths which existed in the devine mind before the morning stars sang together, and which will continue to exist there when the last of their radiant host shal have fallen from heaven*». (Hersh 1999: 9)

Chapter 2 – an outline his project

This chapter describes the research methodology, while a more concrete description of the methodoly is to be found in Appendix A. As explained in the second foreword, Witz has developed a new methodology. This chapter gives a brief account of all the interviews, with students of teacher education, traditional mathematics, and technology. These are interviews that goes in depth, where a grounded theory approach will uncover deeper thoughts. Gratification is the key word, and the intention of this chapter is to show what was uncovered. One student discovered that she really *learned* something, which made her studies much easier.

As a method to collect all his data, Witz has taped all the interviews over a long period of time. These recordings has been rapidly transcribed and the analysis of the notes has led to 2–4 main topics that has been the theme for the next interview. A number of sequential interviews in addition to grounded theory has then given a deeper understanding of a student as a person. This also uncovered the bad experience that many students have with mathematics may be linked to poor teachers in the subject.

The chapter gives a very thorough description of how the interviews were carried out in addition to the background to each of the interviees. Witz is writing about an emerging methodoly, and he describes the three phases of the «*participant as ally – essentialist portraiture*» method as **i)** interview, **ii)** develop understanding for the participant's case, and finally **iii)** cross-case discussion. A result of this methodology is to try to express the participants feelings and consciousness to the reader.

Chapter 3 – the teacher education students

The teacher students Amber and Brian are two students that finds *fulfillment*. Amber, a person of a moral-ethic nature, decided early to become a teacher. The interviews with her has left the impression that she is both thoughtful and has a thirst for knowledge. Brian has always enjoyed teaching mathematics, and wanted to work in a school environment. Between the quotations there are expressions that suggests what he really means. He *knows* that work in a school in an administrative position is right for him, and will give him fulfillment. When the reason is to start with a teaching job and end up in a managerial position, one might wonder if it is the *subject* that inspires. These two students do however know what they want.

Chapter 4 – the PhD student Elizabeth

The PhD student Elisabeth, like the following interviewees, is a traditional mathematician. She is a fine young girl, clever in mathematics, she had clever and well-informed teachers, and she realized early that mathematics was her path in life. The breakthrough for realizing this was number theory and topology, while calculus was not a topic to her liking. Elizabeth came from a business orientet family, but became an academical mathematician that worked with research, and she felt that she was *part of* a mathematics department.

Witz mentiones in one of his comments that Fermat's last theorem was unsolved for 200 years (sic!) – but Fermat scribbled the famous theorem in the margin of his copy of Diophantus' Arithmetika in about 1637, and it was finally proved by Andrew Wiles in 1995.

Chapter 5 – the PhD student Frederick

Frederick is 30 years old, and comes from an academic family, and his path to mathematics went through chemistry and physics at the university. He did have a period as a hippie with LSD and magic mushrooms as ingredients, and that may have made him religiously motivated. In parts of his rootless youth he was also a taxi driver in Alaska. His is a non-practising Catholic, but he did have a revelation when it comes to mathematics, and St. Clement was the reason for that revetation. He has always been fascinated by complex analysis, chaos theory and fractales, and the epiphany opened up the world of abstract algebra, dynamic systems and the beautiful proofs of category theory. His main interests now are early Christianity and dynamic systems.

In one of his comments, Klaus Witz draws the conclusion

> *The mere fact that Frederick was deeply interested in Chategory Theory should have been an indication to me that it must have connected with deeper aspects in him that existed in him before the course.* (Witz 2007: 215)

I find this a very bold conclusion, and I am not sure that it uncovers any new knowledge. Witz is in general very thorough, and has gone to the sources to get a proper understanding of chategory theory and fractals.

Frederick has later been drawn towards Catholicism again, and now he regards mathematics as *sacred activities*.

Chapter 6 – the PhD student Janos

An alternative title for this chapter could have been «*The art of bringing beauty into the world*». The PhD-student Janos became interested in mathematics already in the eight grade, and he decided as a new student that he wanted to study abstract algebra. This was evidently something he wanted to work with. On the way in his studies, he became insecure of himself, and left, but he did return after some years.

Literature has for him been equally important as mathematics. He also talks about *epiphanies*, and about doing sacred activities, and he has also had similar revelations regarding poetry.

Chapter 7 – the PhD student Jack

Finally we have the PhD student Jack, and it seems clear from the interviews that his path is different from the other PhD students – he was occupied with mathematics already in kindergarten. Later, computer programming had the same effect on him as number theory, and recurring activities have always been mathematics, computer science and chess. He has also designed and programmed various computer games. To the question «*What is it about mathematics that is so attractive?*», he replies that «*It is the beauty of it!*». His intense interests seems however to lack structure, because he claims that «*If I appreciate something, I study it!*». The nature of higher mathematics involves pure perception. He lost quickly the interest for analysis, because all the counter examples did *not* represent beauty.

He also admits that Ayn Rand has made an impression on his idea of how the world should be.

Chapter 8 – analyzing the interviews

This very important chapter contains the discussion of the interviews, and of the *inner understanding*. The various forewords calls it a key chapter, and the most important chaper in the whole book.

What does the previous chapters conclude with? Elizabeth, Janos and Jack all reveal a tacit inner understanding, or vision, of mathematics and its nature. A very important issue is then the explanation of what the nature, inner understanding and inner vision of mathematics is. Written interviews like the ones in this book may be regarded as he moust basic units in qualitative research, and witz are mentioning some objections.

There are of course a vast number of examples through history to exemplify the ideas Witz is emphasizing. When Brouwer gave a lecture about geometry in *four* dimensions, he needed an *inner understanding* and this understanding is a *pure conception*. Paul Erdös has said that mathematics is the only human *infinite* activity. When mathematiciens meet, it looks like they recognize a common set of values, consciousness or spirit. The *inner understanding* makes one able to rise oneself above fragmentation, and to see things as a whole. Is this inner understanding something that is reserved to scholarly people at a level of PhD-student and up? – and is it something that is exclusive for mathematics, and does not apply for other subjects?

Chapter 9 – some reflections

The points from the previous chapter are now viewed in a broader perspective. It is an ideal situation that one sees a greater coherence, and then becomes a part of it. *Bildung* is key issue, and has for over 200 years been the main ideal for all educaion in Europe, but the influence has faded since the 1960'ies. Wilhelm von Humboldt claimed that all Bildung is in the innermost sole. Witz is making clear references to the concept of *Bilding*, and as he writes that «*the portraits illustrate not only that mathematics may play a larger role in forming the student. They also affirm the* Bilding *theory vision of the individual*» (Witz 2007: 369). The portraits are illustrations of the «*vision of the individuals*» from the Bildung theory.

Appendices – methodology and some mathematical topics

Appendix A may well be regarded as a method chapter for the book. The human being as a *whole* is goal for the research, and four traditions in qualitative research are described thoroughly – *phenomenological*, *biographical* and *narrative* research, as well as case-studies, particularly the *Albert-Murray* tradition of biographical case-studies. Witz claims the position of *post-positivistic* – somewhere between positivism and postmodernism – and the method is described as

1. *something is being studied*
2. *interaction with a participant*
3. *interaction is being recorded, i.e. taped*
4. *the data is being analyzed*
5. *understanding, analysis and data is being published*

The method is epistemologically based on Cooley's «*principle of sympathetic introspection*» – a deeper understanding of another person emerges by placing oneself in close contact with the other person, and allowing the other person to wake in oneself a life equivalent to ones own, and then afterwards, to the best of our abilities, to recall and describe. This, Witz asserts, gives a possibility to understand the other's consciousness.

The appendices *B* through *D* gives a useful description of the mathematical subject matter referred to by the various interviewees, and finally *Appendix E* contains an elaboration of the concepts that is crucial to the book, and especially chapter 8. «*Inner vision*» is a result of *awakening* and *encounter*, and «*crystallizing experiences*» are the first sprouts that crystallizes – the interest awakens.

Methodological issues

The constructivist's view says that *there is no discovery learning* in the sense of any unmasking of external structures. The arrival of new knowledge is based on a creative combination of knowledge already available to the student. In Piagetan terms this is called *accommodation* (Bauersfeld 1995), and a lonely child will *not*, according to Bauersfeld, develop. The socio-cultural theorist Vygotsky writes, on the other hand, about an internalizing of knowledge which is dependent of what the child already knows. Learning is, according to Vygotsky's approach, something that first occurs *between* persons, before a person *internalizes* the knowledge and makes it ones own, or *intrapersonal.* This process consists of a series of transformations where external activities are reconstructed and begin to occur internally, then an interpersonal process is transformed into an intrapersonal one through a series of development events. The laws defining the activities of this process are incorporated into a new system with new laws (Vygotsky 1978). I have no answer, but is it possible to explain the interviewed students in this book in these two models? Some of them really looks like lone children that has developed.

The term *hermeneutics* covers the theory of understanding and interpretation of linguistic and non-linguistic expressions. The *hermeneutic circle* is a central idea of all hermeneutic thinking, and it is viewed in terms of explanatory interdependency, between the parts and the whole of a meaningful structure. Gadamer (2007) uses the word *prejudice* in a positive way, and our first approach to the text is our prejudices. As we move in the hermeneutic circle our prejudices are replaced with more and more understanding, as our understanding is given over to that which is to be understood. Gadamer is occupied with role prejudice plays in our understanding. Prejudice or pre-understanding comes from the fact that we are caught in a tradition we cannot liberate ourselves from, and this tradition influences the way we interprets texts.

This book has been studied using the principles of the *hermeneutic circle*, also called the *hermeneutic spiral.* The latter implies that we are not repeating ourselves by going in circles, but we are actually getting deeper into the text by using this method. We continue until using new approaches no longer produces any interesting findings. It is also possible to alternate between two views – a *global* view of the books, and a *detailed* view into the various interviews. Gadamer (2007) introduces the concept of *horizons* when reading a text. The reader has a horizon, and text has a horizon of its own, and when we are trying to fuse together these two horizons, a new horizon will arise. This fusion is the aim of the hermeneutics.

Some concluding remarks

There are some unfortunate mishaps that may challenge the reader of Witz' book. On page 2 one can read «*(see Table 2.1 below)*», but the table appears first on page 49. Lincoln and Guba (1994) is missing from the bibliography, and Walters and Gardner has the years 1984 og 1993 in the text, and the year 1986 in the bibliography. There are lots of useful notes in the book, but they are at the end of each chapter; they would have been easier to find if they were at the bottom of each page, or at the end of the book. This is a book that one want to read in hermeneutic circles, but the layout makes this a bit difficult as there is nothing on a page telling you which chapter you are in.

The book is very long, and what makes it long is all the quotations from the interviews and the reflections upon them. At the same time, I would not have missed any of them. It is the interviews and the in-depth reflections that makes this book a readable book for people who wants to know more about what goes on when some young people get inspired by mathematical ideas and concepts. Literature, music and pictorial art are considered as art forms by everyone. When regarding *mathematics* as an art form, one have to understand the notation, etc. In mathematics, as in literature, but not in music, the interpreter and the public is one and the same person.

One cannot, in an essay about inner visions and innner understanding in mathematics, avoid mentioning the brilliant indian mathematician *Srinivasa A. Ramanujan (1887–1920)*. He sent letters with his mathematical findings to several well-known mathematicians, who ingrored him, untill he finally sent his letters to G.H. Hardy, who did not ingore him. Hardy and Ramanujan began a renowned cooperation, and Ramanujans letters and formulas strongly indicate that there is mathematical intuition. Ramanujan did not include any proofs, but all his formulas and theorems were correct, and there were over a hundred of them. This is reliable mathematical belief without being formalized. In his first letter to Hardy, Ramanujan introduces himself as «*a clerk in the Accounts Department of the Port Trust Office at Madras*» (Berndt and Rankin 1991; Hersh 1999). Ramanujan was truly inspired, and he was discovered by a mathematician, also known for his love and admiration to the field.

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